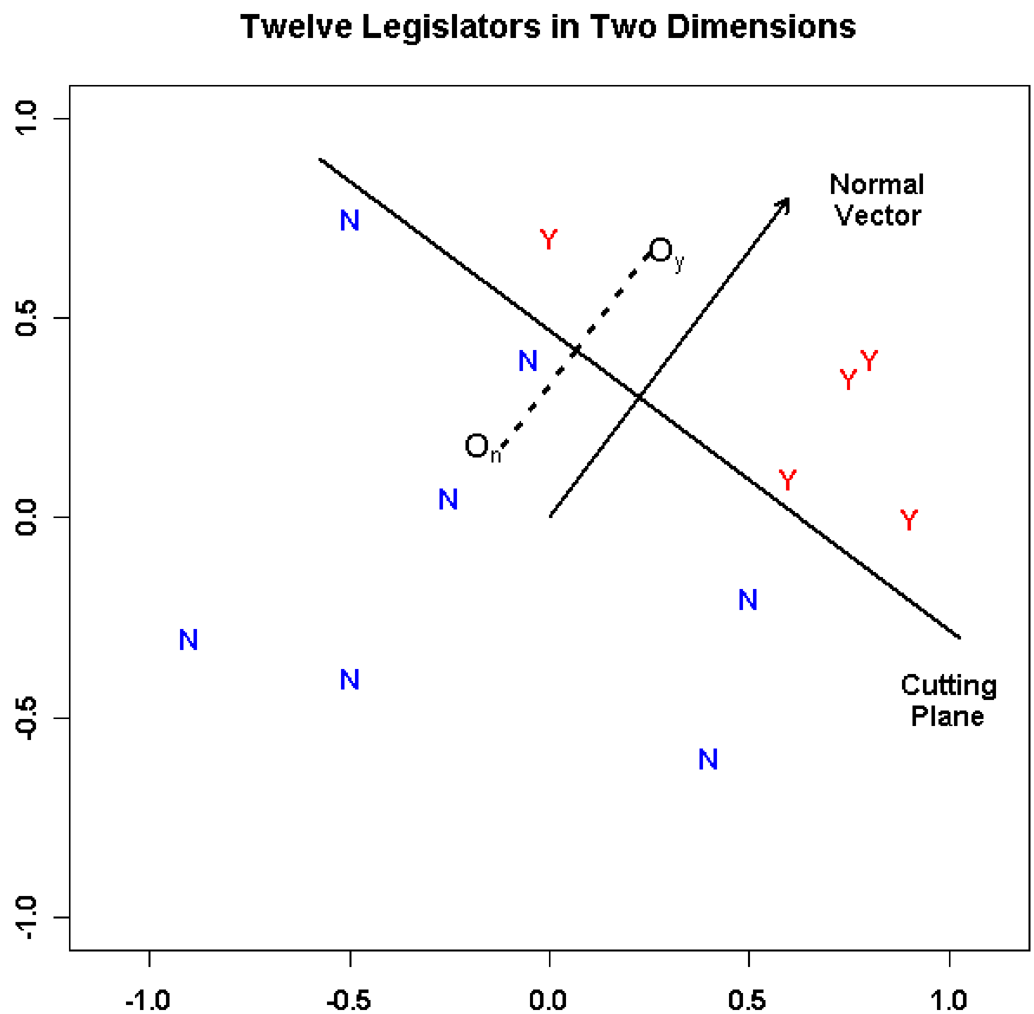
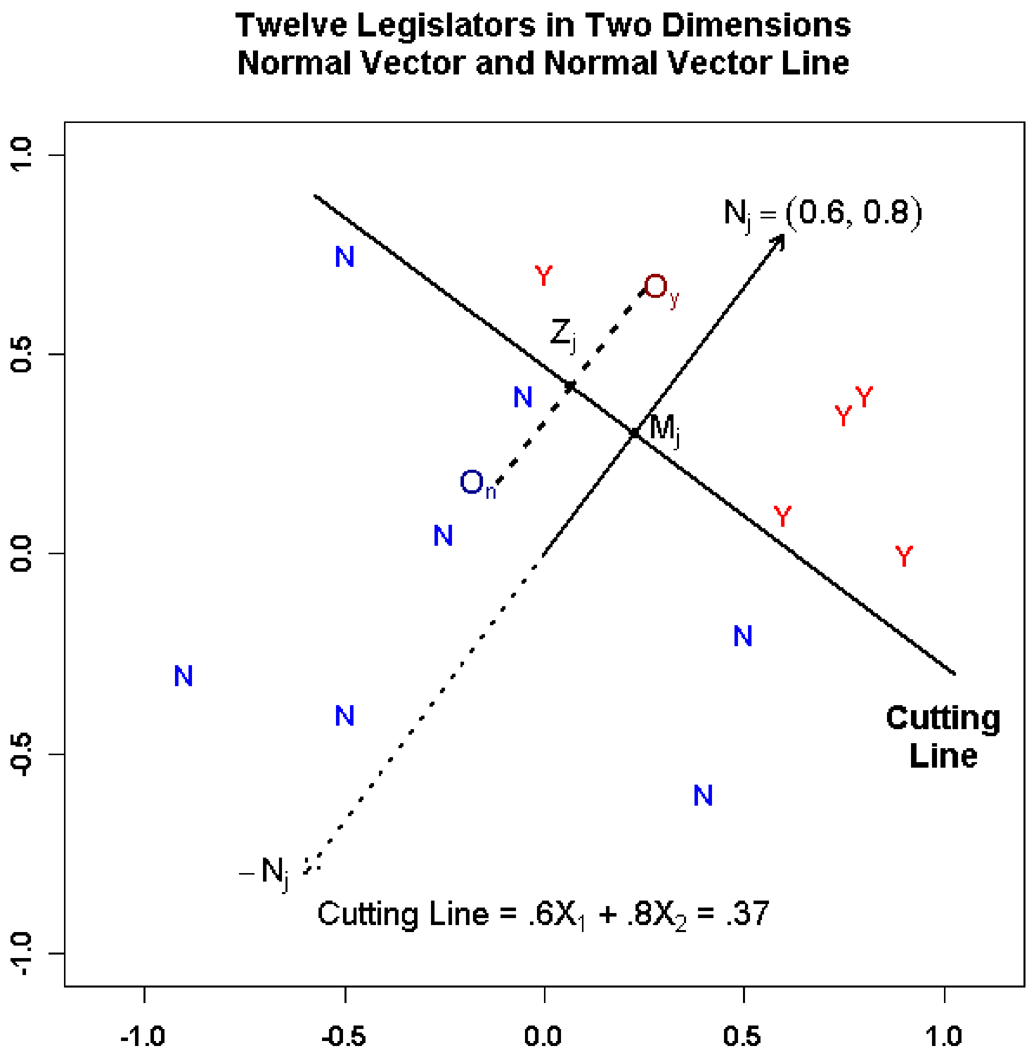
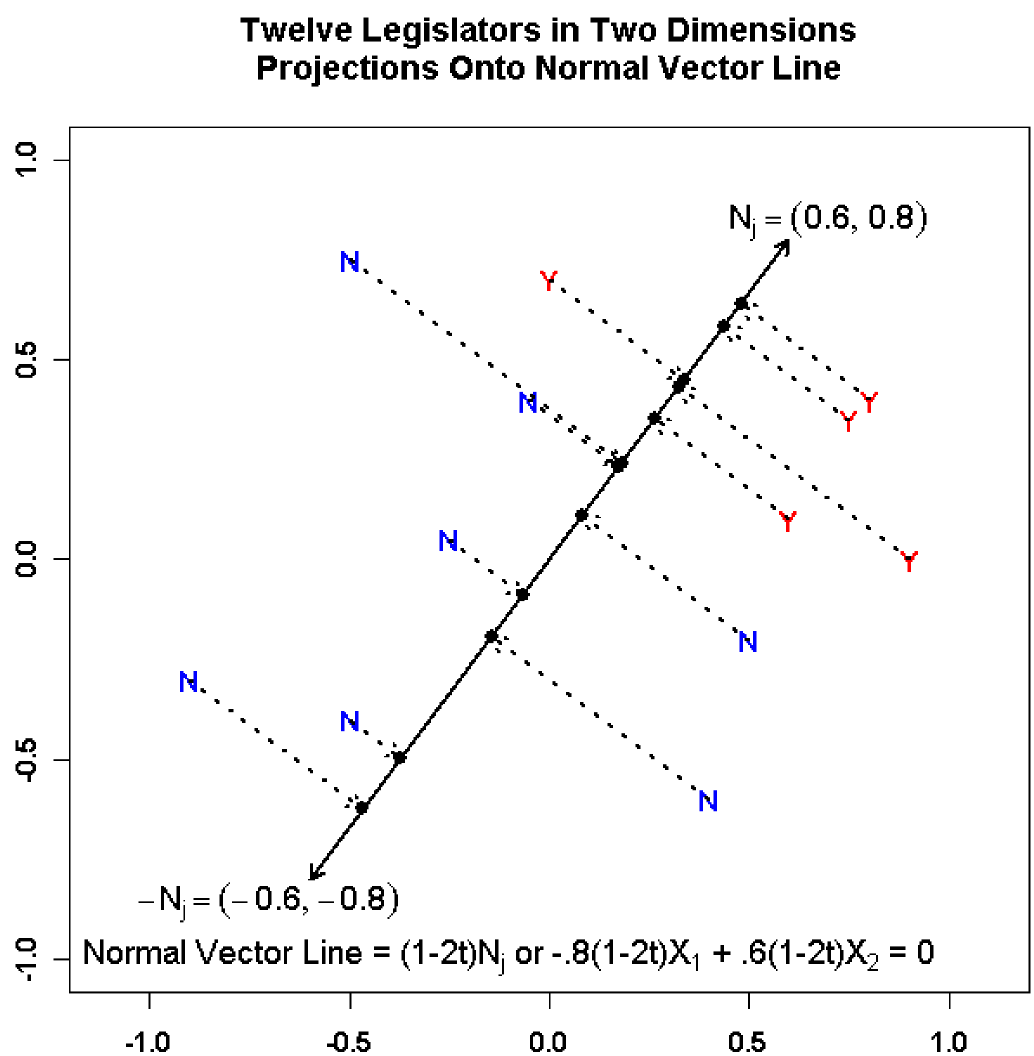
The Programs that created these figures are:

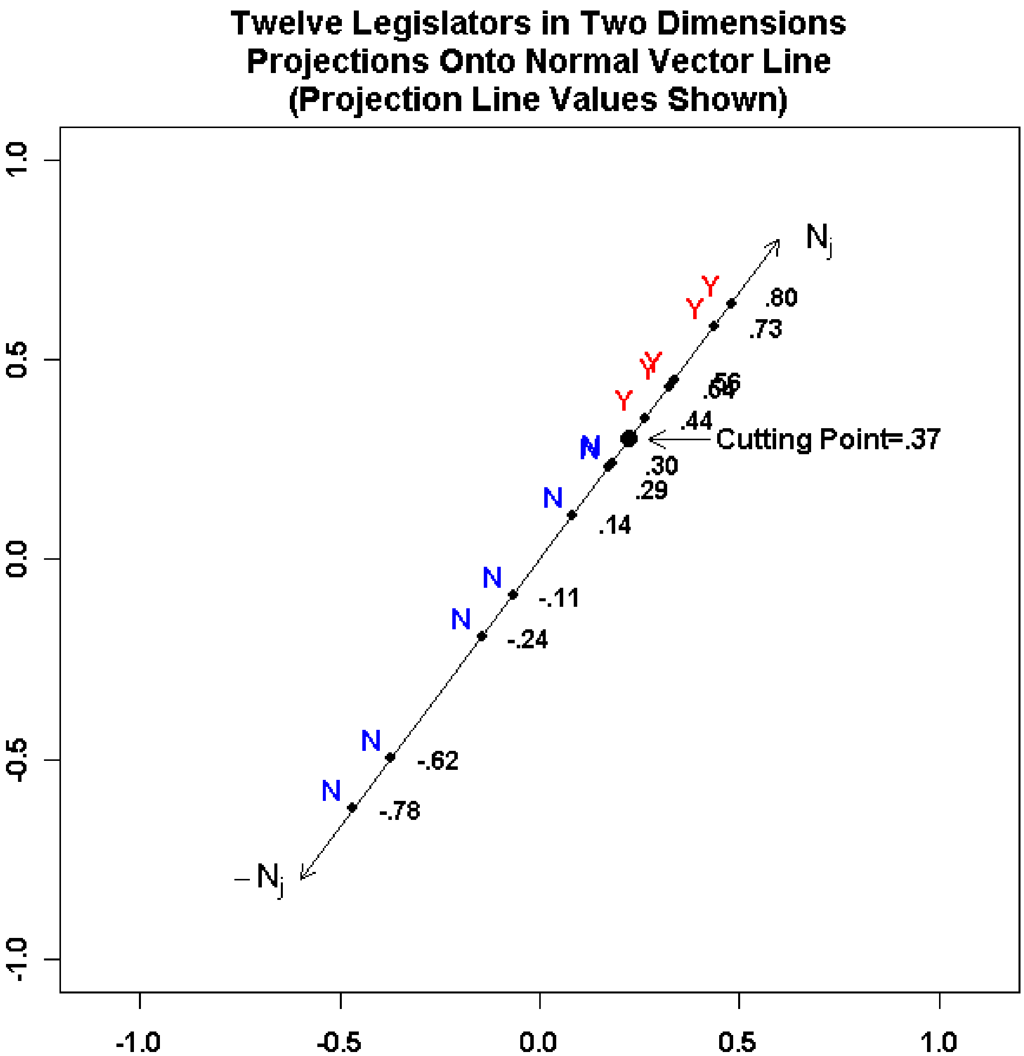
```
p.2  book_ch_2_fig_7_final.r
p.3  book_ch_2_fig_11.r
p.4  figure_3_handout_2D_geometric_2008.r
p.5  book_ch_3_fig_8b.r
p.6  poole_chap03_fig09.doc
p.7  book_ch_3_fig_10A_final.r
p.8  book_ch_3_fig_10B_final.r
p.9  book_ch_3_fig10C.r
p.10 book_ch_3_fig_10_OLS_projection.r
p.11 book_ch_3_fig10C_projection.r
p.12 book_ch_3_fig10C_on_line.r
p.13 book_ch_3_fig12B_on_line.r
p.14 book_ch_3_fig12C_on_line.r
p.15-17 book_ch_3_fig13_final.r
p.18 poole_chap03_table01.doc
```


Twelve Legislators in Two Dimensions
Normal
Vector
Cutting
Plane
O_y
O_n
N
N
N
N
N
N
N
Y
Y
Y
Y
Y
1.0
0.5
0.0
-0.5
-1.0
-1.0
-0.5
0.0
0.5
1.0


Twelve Legislators in Two Dimensions
Normal Vector and Normal Vector Line
N
Y
N
Y
Y
N
Y
N
Y
N
N
N
N
N
O_n
O_y
Z_j
M_j
N_j = (0.6, 0.8)
– N_j
Cutting Line
Cutting Line = .6X_1 + .8X_2 = .37
1.0
0.5
0.0
-0.5
-1.0
-1.0
-0.5
0.0
0.5
1.0



Twelve Legislators in Two Dimensions
Projections Onto Normal Vector Line
$N_j = (0.6, 0.8)$
$-N_j = (-0.6, -0.8)$
Normal Vector Line = $(1-2t)N_j$ or $-.8(1-2t)X_1 + .6(1-2t)X_2 = 0$
-1.0
-0.5
0.0
0.5
1.0

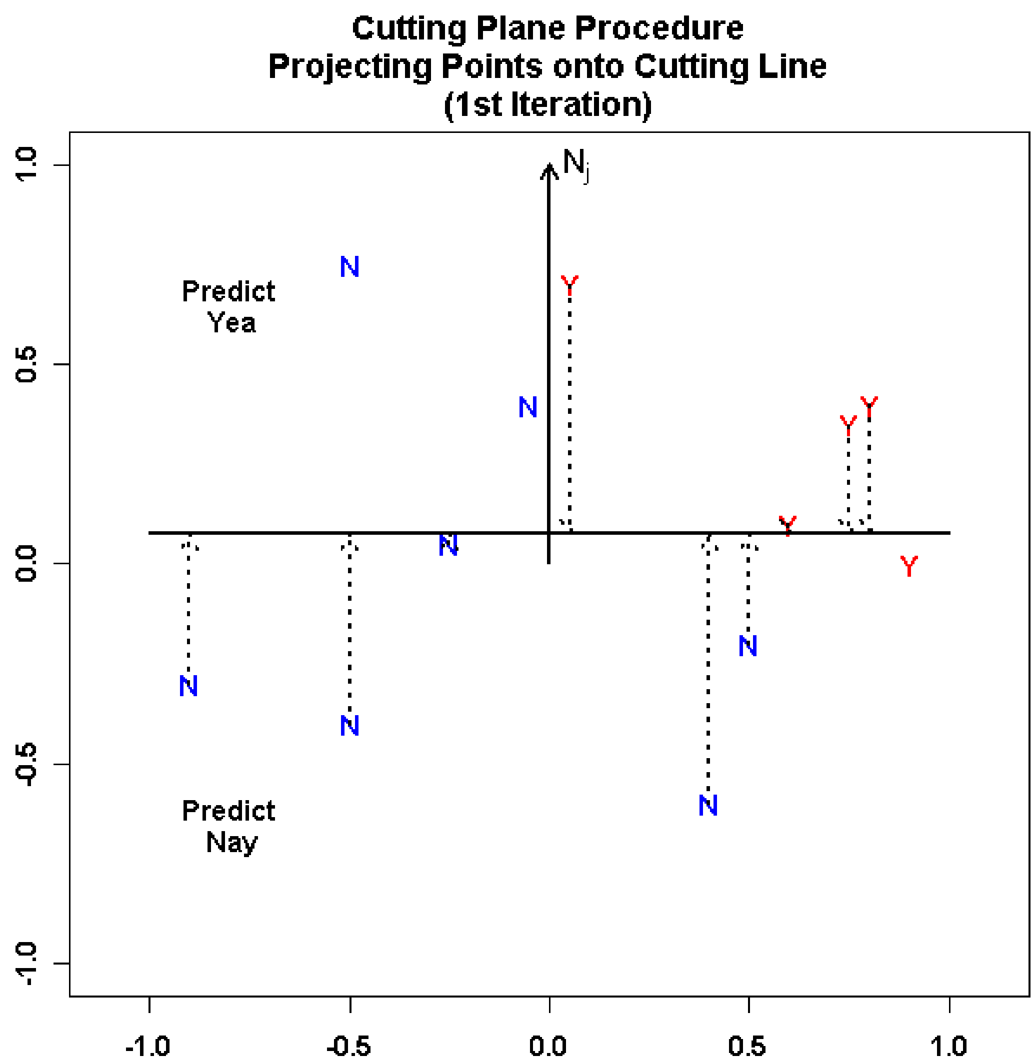

Twelve Legislators in Two Dimensions
Projections Onto Normal Vector Line
(Projection Line Values Shown)
N_j
$-N_j$
Cutting Point=.37
.80
.73
.56
.54
.44
.30
.29
.14
-.11
-.24
-.62
-.78
Y
N
-1.0
-0.5
0.0
0.5
1.0

Legislator Points From Figure 3.8 | **Legislator Points Projected Onto Normal Vector Line**

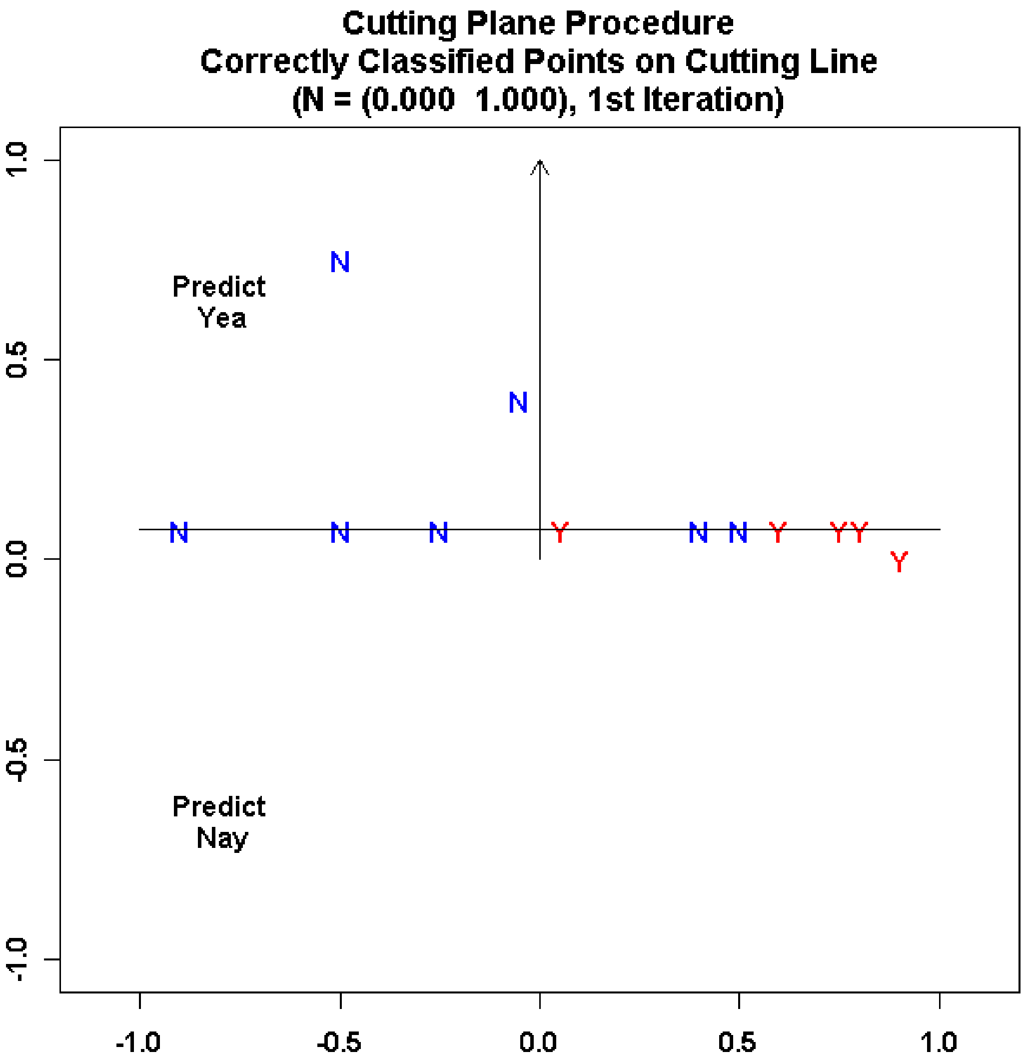
$$\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \\ x_{41} & x_{42} \\ x_{51} & x_{52} \\ x_{61} & x_{62} \\ x_{71} & x_{72} \\ x_{81} & x_{82} \\ x_{91} & x_{92} \\ x_{101} & x_{102} \\ x_{111} & x_{112} \\ x_{121} & x_{122} \end{bmatrix} = \begin{bmatrix} -.90 & -.30 \\ -.50 & -.40 \\ .40 & -.60 \\ -.25 & .05 \\ .50 & -.20 \\ -.05 & .40 \\ -.50 & .75 \\ .60 & .10 \\ .90 & .00 \\ .00 & .70 \\ .75 & .35 \\ .80 & .40 \end{bmatrix} \qquad \begin{bmatrix} -.468 & -.624 \\ -.372 & -.496 \\ -.144 & -.192 \\ -.066 & -.088 \\ .084 & .112 \\ .174 & .232 \\ .180 & .240 \\ .264 & .352 \\ .324 & .432 \\ .336 & .448 \\ .438 & .584 \\ .480 & .640 \end{bmatrix}$$

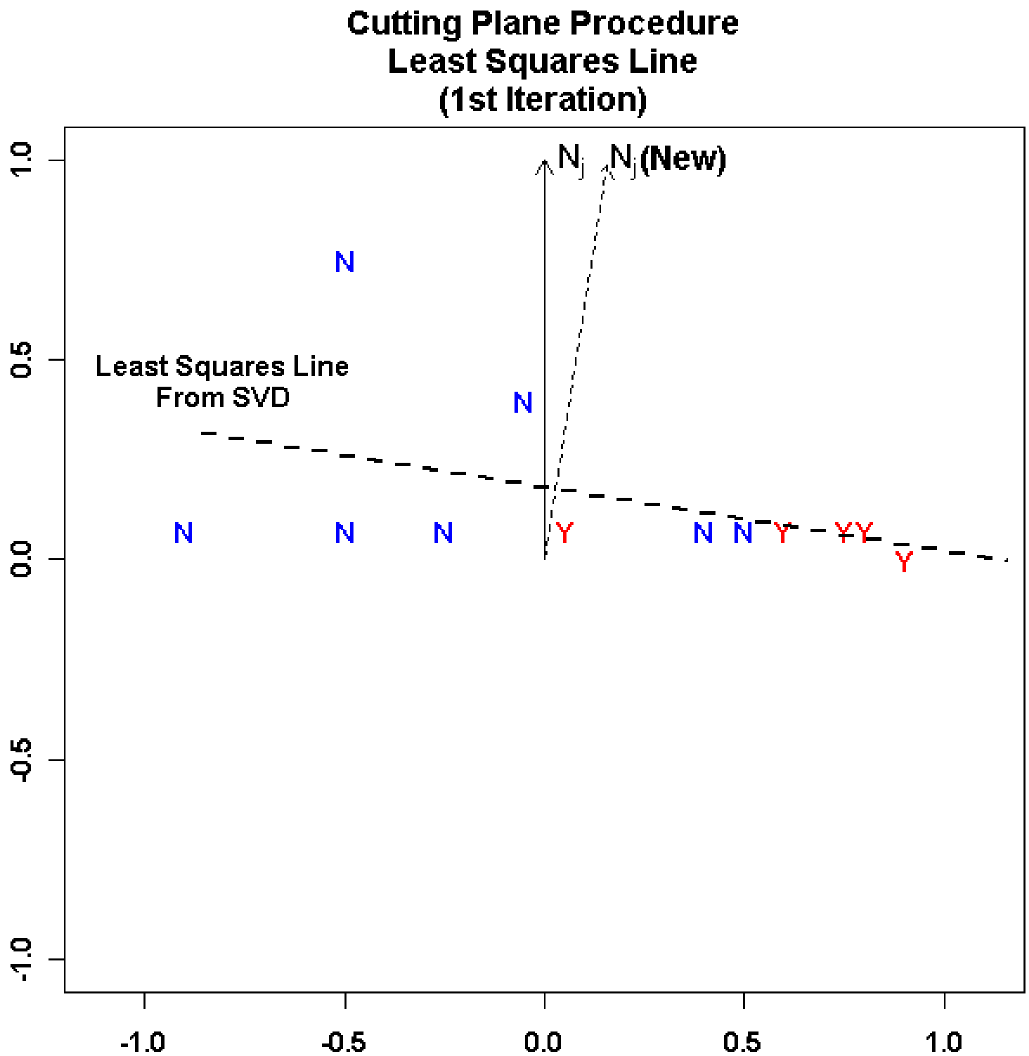
Legislator Values on *Projection Line*: $X_i'N_j = w_i$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \\ w_9 \\ w_{10} \\ w_{11} \\ w_{12} \end{bmatrix} = XN_j = \begin{bmatrix} -.90 & -.30 \\ -.50 & -.40 \\ .40 & -.60 \\ -.25 & .05 \\ .50 & -.20 \\ -.05 & .40 \\ -.50 & .75 \\ .60 & .10 \\ .90 & .00 \\ .00 & .70 \\ .75 & .35 \\ .80 & .40 \end{bmatrix} \begin{bmatrix} .6 \\ .8 \end{bmatrix} = \begin{bmatrix} -.78 \\ -.62 \\ -.24 \\ -.11 \\ .14 \\ .29 \\ .30 \\ .44 \\ .54 \\ .56 \\ .73 \\ .80 \end{bmatrix}$$

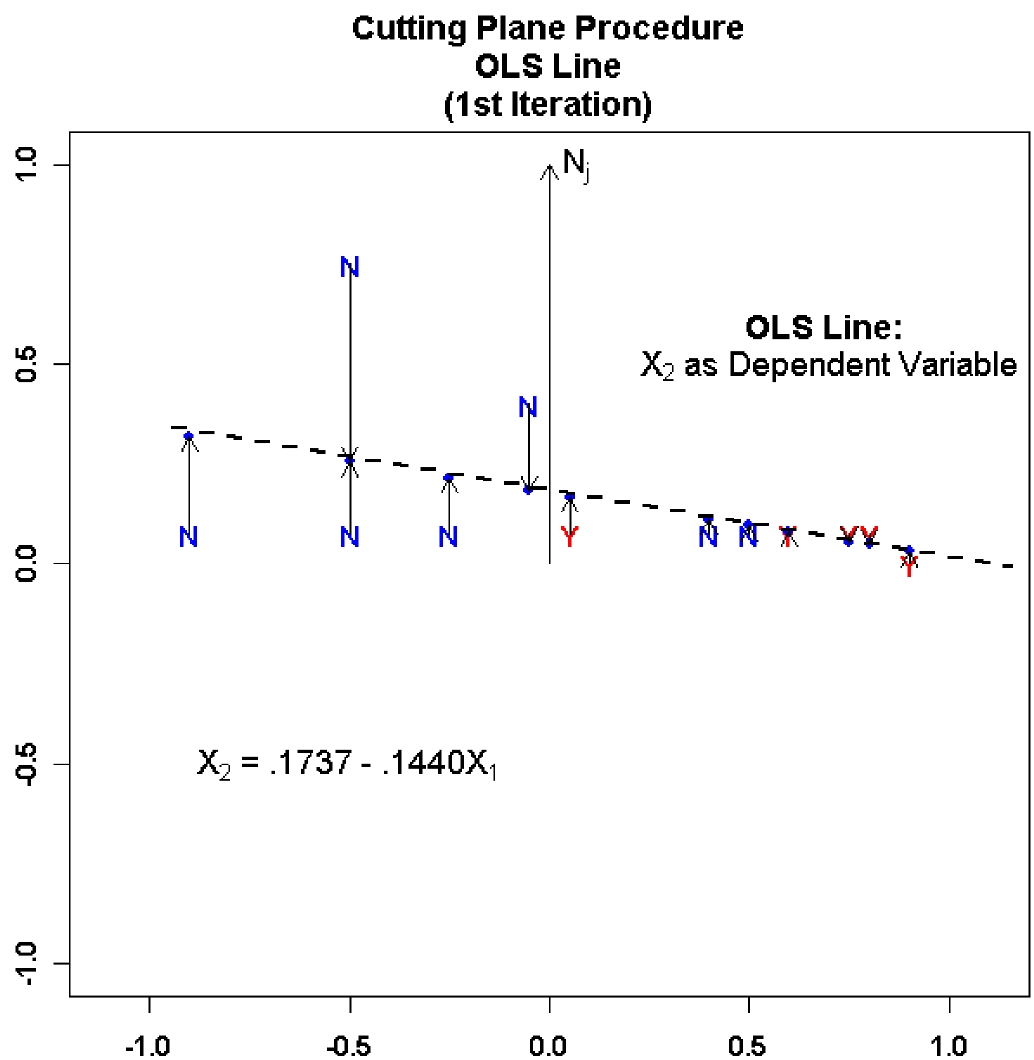


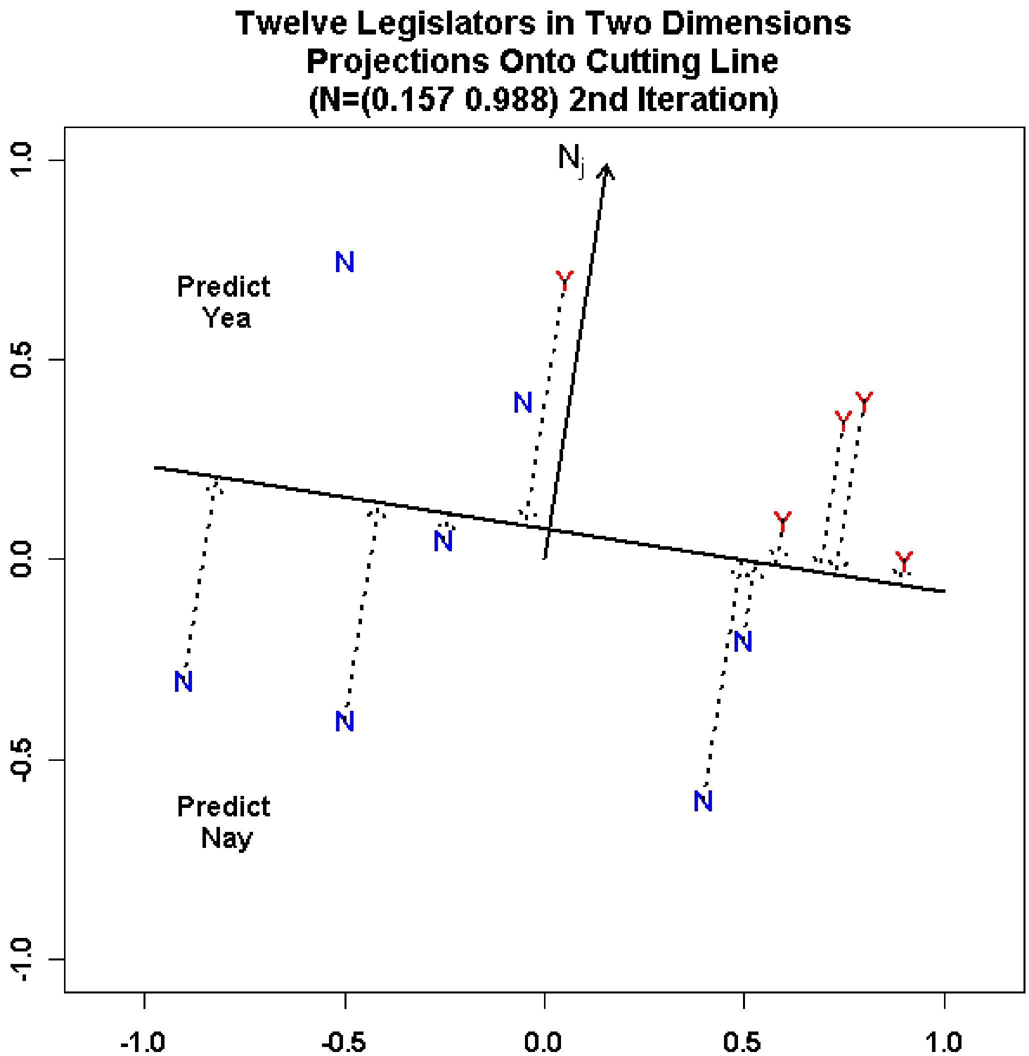
Cutting Plane Procedure
Projecting Points onto Cutting Line
(1st Iteration)
Predict
Yea
Predict
Nay
N
Y
1.0
0.5
0.0
-0.5
-1.0
-1.0
-0.5
0.0
0.5
1.0

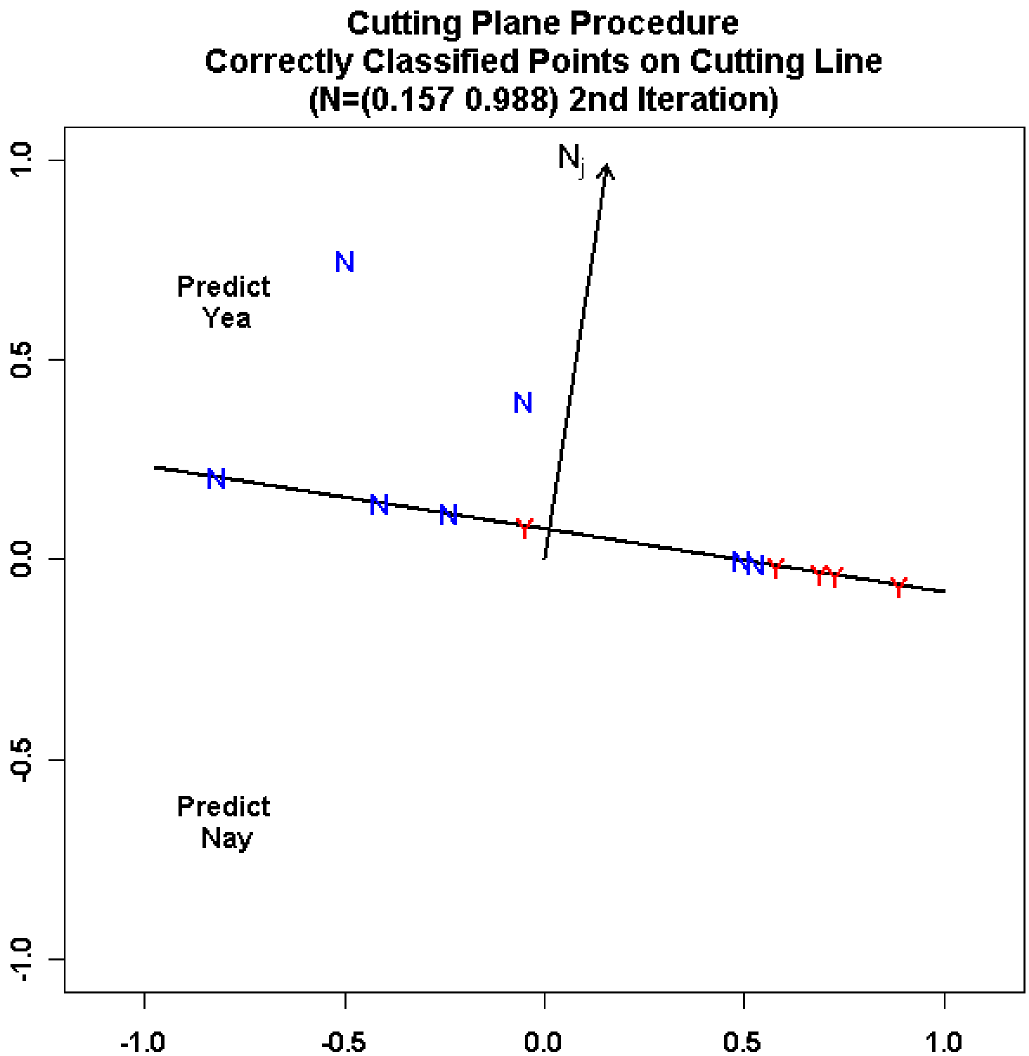

Cutting Plane Procedure
Correctly Classified Points on Cutting Line
(N = (0.000 1.000), 1st Iteration)
Predict
Yea
Predict
Nay
N
N
N
N
N
N
N
Y
Y
Y
Y
Y
1.0
0.5
0.0
-0.5
-1.0
-1.0
-0.5
0.0
0.5
1.0

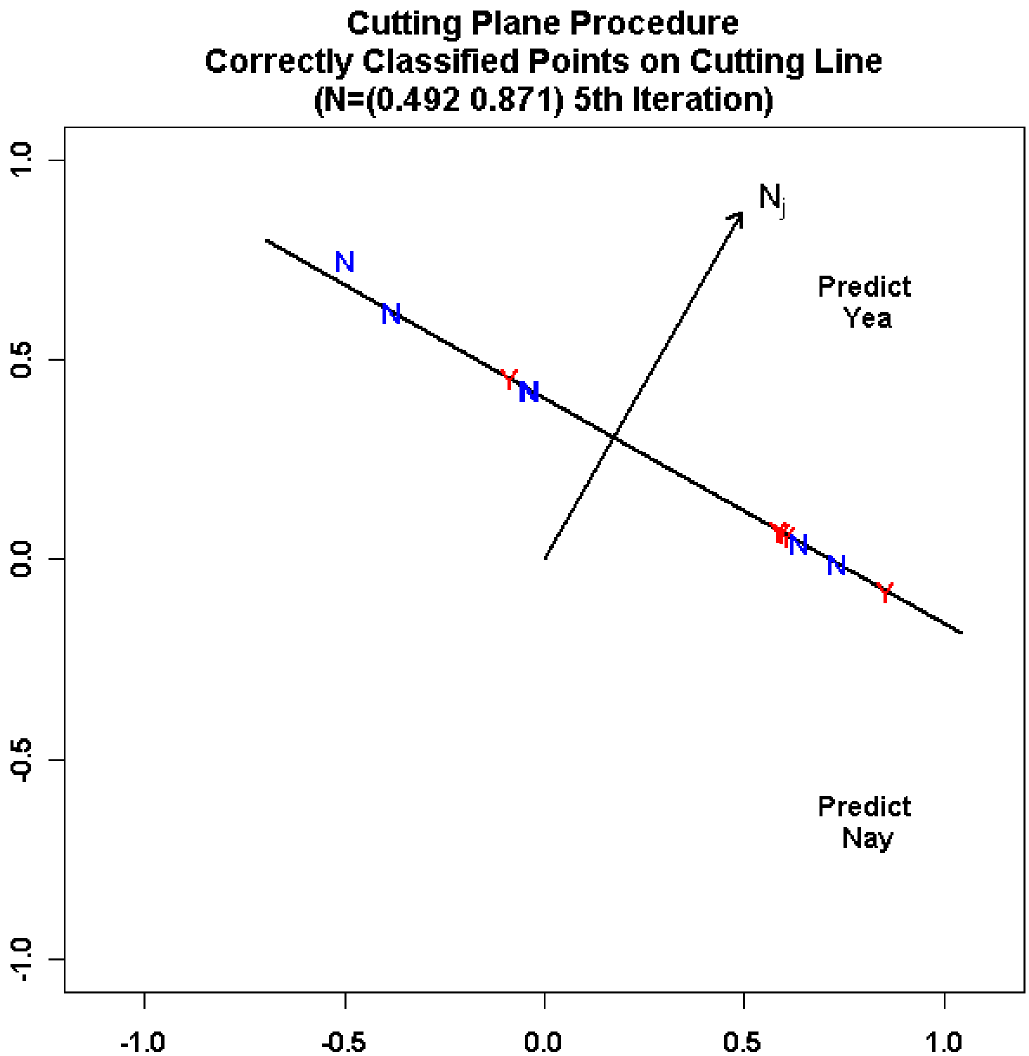


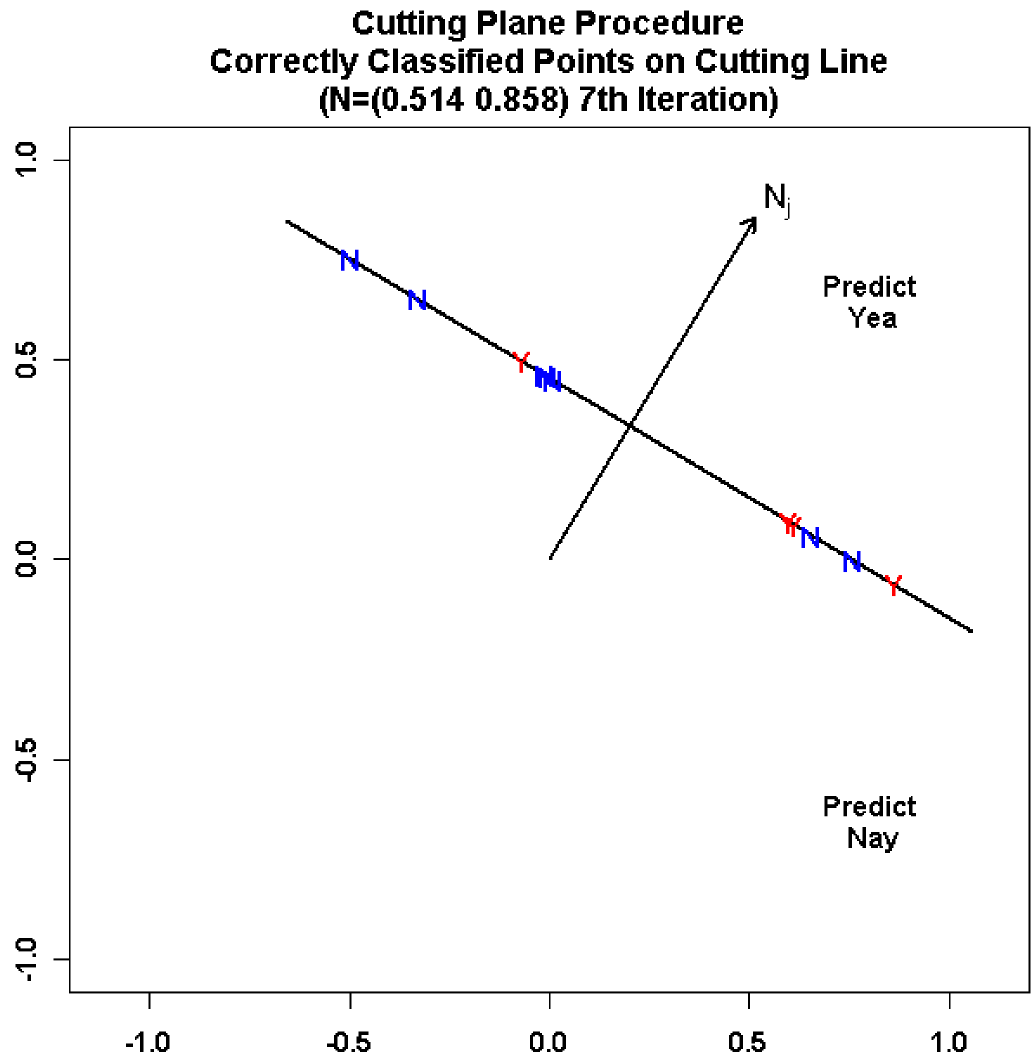
Cutting Plane Procedure
Least Squares Line
(1st Iteration)
Nj
Nj (New)
Least Squares Line
From SVD
N
Y
1.0
0.5
0.0
-0.5
-1.0

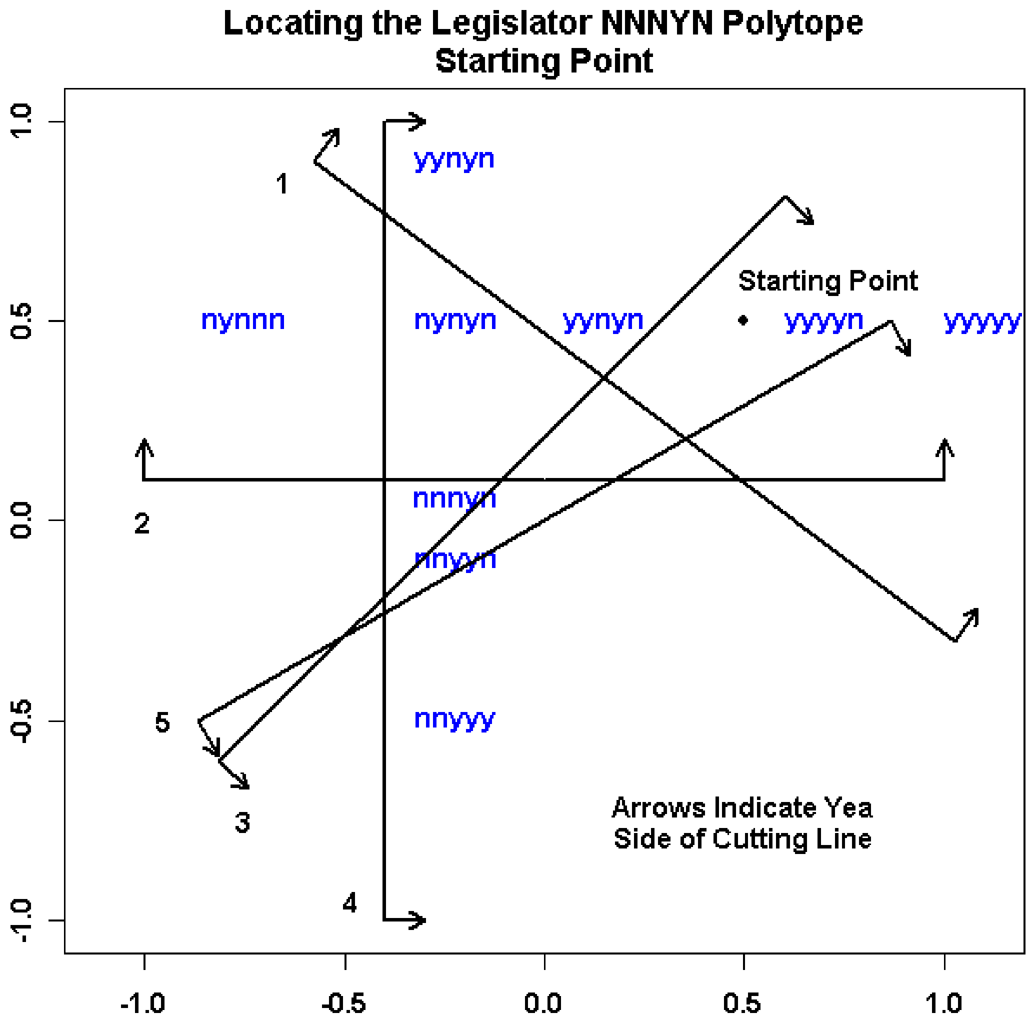

Cutting Plane Procedure
OLS Line
(1st Iteration)
N_j
OLS Line:
X2 as Dependent Variable
X2 = .1737 - .1440X1
N
Y
1.0
0.5
0.0
-0.5
-1.0

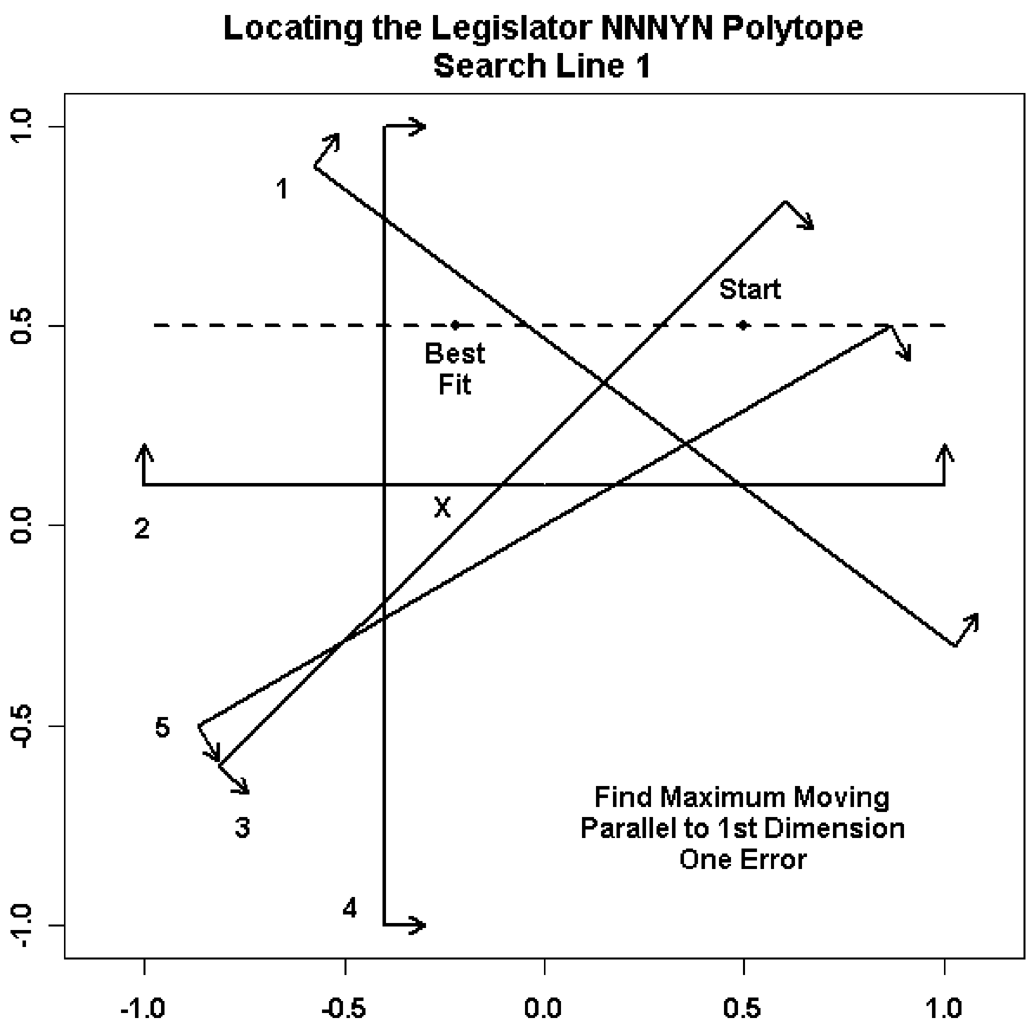

Twelve Legislators in Two Dimensions
Projections Onto Cutting Line
(N=(0.157 0.988) 2nd Iteration)
Predict
Yea
Predict
Nay
Nj
-1.0
-0.5
0.0
0.5
1.0


Cutting Plane Procedure
Correctly Classified Points on Cutting Line
(N=(0.157 0.988) 2nd Iteration)
Nj
Predict
Yea
Predict
Nay
N
N
N
N
N
Y
NNY
YY
Y
1.0
0.5
0.0
-0.5
-1.0
-1.0
-0.5
0.0
0.5
1.0


Cutting Plane Procedure
Correctly Classified Points on Cutting Line
(N=(0.492 0.871) 5th Iteration)
Nj
Predict
Yea
Predict
Nay
N
N
Y
N
Y
N
N
Y
1.0
0.5
0.0
-0.5
-1.0
-1.0
-0.5
0.0
0.5
1.0


Cutting Plane Procedure
Correctly Classified Points on Cutting Line
(N=(0.514 0.858) 7th Iteration)
Nj
Predict
Yea
Predict
Nay
-1.0
-0.5
0.0
0.5
1.0


Locating the Legislator NNNYN Polytope
Starting Point
1
yynyn
Starting Point
nynnn
nynyn
yynyn
yyyyn
yyyyy
2
nnnyn
nnyyn
5
nnyyy
3
Arrows Indicate Yea
Side of Cutting Line
4
1.0
0.5
0.0
-0.5
-1.0
-1.0
-0.5
0.0
0.5
1.0


Locating the Legislator NNNYN Polytope
Search Line 1
1.0
0.5
0.0
-0.5
-1.0
1
2
3
4
5
Start
Best
Fit
X
Find Maximum Moving
Parallel to 1st Dimension
One Error
-1.0
-0.5
0.0
0.5
1.0

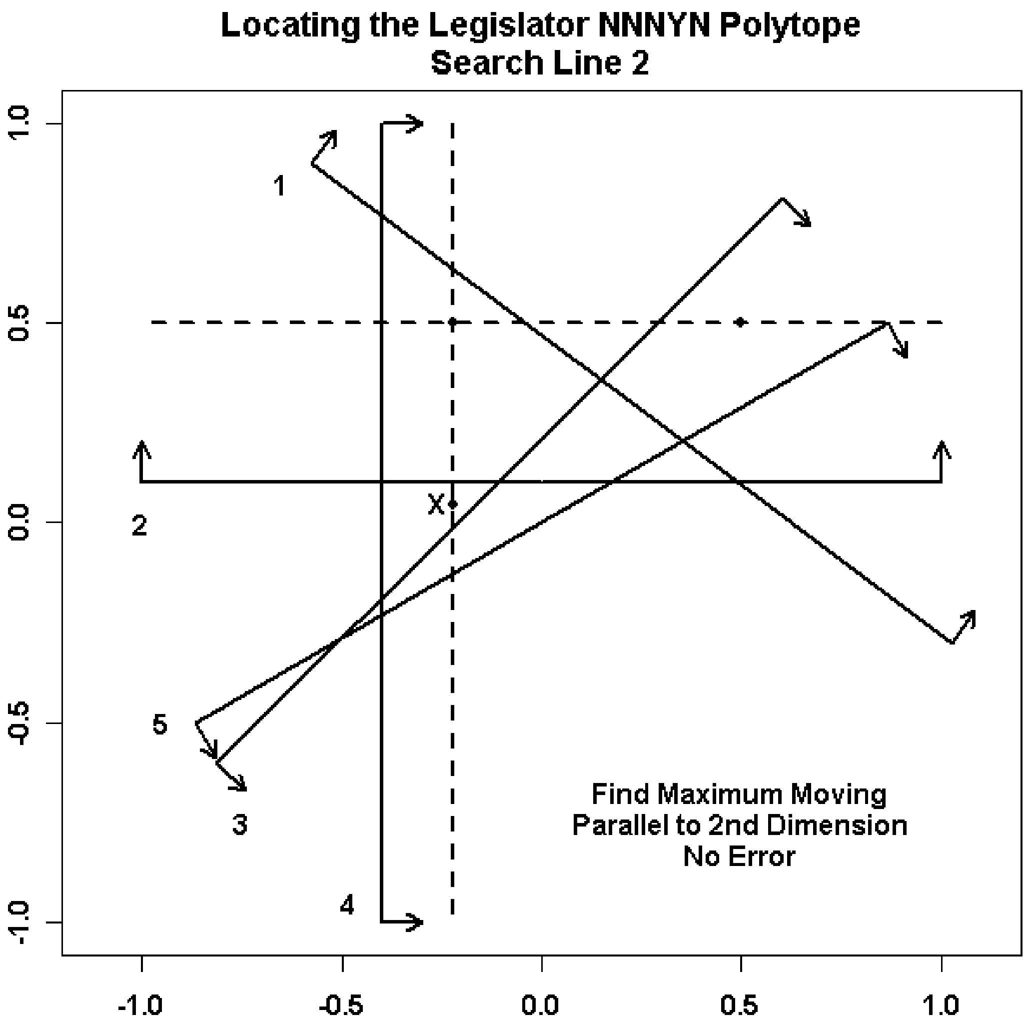

Locating the Legislator NNNYN Polytope
Search Line 2
1.0
0.5
0.0
-0.5
-1.0
1
2
3
4
5
X
Find Maximum Moving
Parallel to 2nd Dimension
No Error
-1.0
-0.5
0.0
0.5
1.0

Table 3.1 Twelve Possible Orderings of Legislator and Cutting Point

Case	Ordering	Classification h	Classification a	Limits of α That Correctly Project $x_i^{(h+1)}$
1.	$-R < c_j < w_{ij}^{(h)} < w_{ij}^{(a)} < +R$	C[1]	C	$\frac{c_j - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}} < \alpha_j < \frac{R - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}}$
2.	$-R < c_j < w_{ij}^{(h)} < w_{ij}^{(a)} < +R$	I	I	$\frac{-R - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}} < \alpha_j < \frac{c_j - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}}$
3.	$-R < c_j < w_{ij}^{(a)} < w_{ij}^{(h)} < +R$	C	C	$\frac{R - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}} < \alpha_j < \frac{c_j - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}}$
4.	$-R < c_j < w_{ij}^{(a)} < w_{ij}^{(h)} < +R$	I	I	$\frac{c_j - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}} < \alpha_j < \frac{-R - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}}$
5.	$-R < w_{ij}^{(h)} < w_{ij}^{(a)} < c_j < +R$	C	C	$\frac{-R - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}} < \alpha_j < \frac{c_j - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}}$
6.	$-R < w_{ij}^{(h)} < w_{ij}^{(a)} < c_j < +R$	I	I	$\frac{c_j - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}} < \alpha_j < \frac{R - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}}$
7.	$-R < w_{ij}^{(a)} < w_{ij}^{(h)} < c_j < +R$	C	C	$\frac{c_j - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}} < \alpha_j < \frac{-R - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}}$
8.	$-R < w_{ij}^{(a)} < w_{ij}^{(h)} < c_j < +R$	I	I	$\frac{R - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}} < \alpha_j < \frac{c_j - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}}$
9.	$-R < w_{ij}^{(h)} < c_j < w_{ij}^{(a)} < +R$	C	I	$\frac{-R - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}} < \alpha_j < \frac{c_j - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}}$
10.	$-R < w_{ij}^{(h)} < c_j < w_{ij}^{(a)} < +R$	I	C	$\frac{c_j - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}} < \alpha_j < \frac{R - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}}$
11.	$-R < w_{ij}^{(a)} < c_j < w_{ij}^{(h)} < +R$	C	I	$\frac{R - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}} < \alpha_j < \frac{c_j - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}}$
12.	$-R < w_{ij}^{(a)} < c_j < w_{ij}^{(h)} < +R$	I	C	$\frac{c_j - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}} < \alpha_j < \frac{-R - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}}$

[1] "C" is correctly classified; "I" is incorrectly classified.

$$\mathbf{X_i}^{(h+1)} = \mathbf{X_i}^{(h)} + \alpha(\mathbf{X_i}^{(a)} - \mathbf{X_i}^{(h)}) \qquad (3.11)$$

Projected onto the j^{th} normal vector, becomes:

$$w_{ij}^{(h+1)} = w_{ij}^{(h)} + \alpha(w_{ij}^{(a)} - w_{ij}^{(h)}) \qquad (3.12)$$

For a single roll call, it is easy to solve for α (see Table 3.1)

For case 2, α must be chosen so that the projection of

$\mathbf{X_i}^{(h+1)}$, $w_{ij}^{(h+1)}$, is in the region $(-R, c_j)$. Solving for α:

$-R = w_{ij}^{(h)} + \alpha(w_{ij}^{(a)} - w_{ij}^{(h)})$ so that

$$\frac{-R - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}} < \alpha_j$$

$c_j = w_{ij}^{(h)} + \alpha(w_{ij}^{(a)} - w_{ij}^{(h)})$ so that

$$\alpha_j < \frac{c_j - w_{ij}^{(h)}}{w_{ij}^{(a)} - w_{ij}^{(h)}}$$