

Euclid's Postulates

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.
5. Parallel Postulate: For every line l and for every point P that does not lie on l there exists a unique line m through P that is parallel to l .

The entire edifice of probability theory is generated from these 3 axioms:

axiom 1: All probabilities lie between 0 and 1.

axiom 2: The probability of the Sample Space is 1.

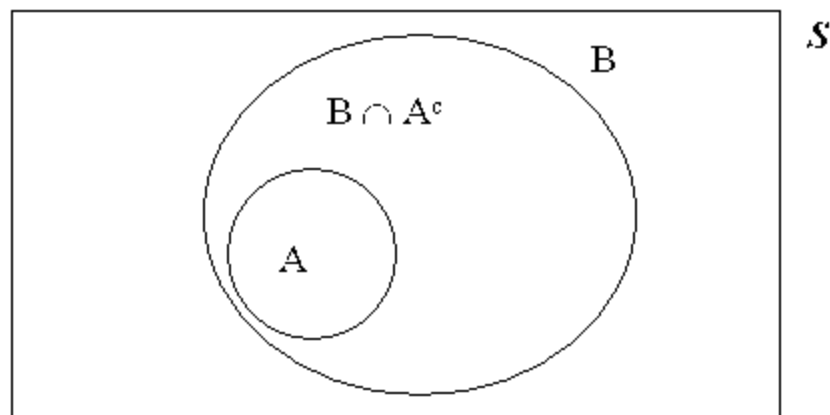
axiom 3: The probability of the union of mutually exclusive events is the sum of their separate probabilities.

The probability of the null set is 0 and the probability of an event is equal to 1 minus the probability of the complement of the event; that is,

$$P(\emptyset) = 0 = 1 - P(S), \text{ and}$$

$$P(A) = 1 - P(A^c).$$

If $A \subset B$, then $P(A) \leq P(B)$. This is easily demonstrated with a simple Venn Diagram.



The "trick" is to see that the set B can be written as the union of the mutually exclusive sets A and $B \cap A^c$. By applying axiom 3 it is easy to show the truth of the statement. Namely,

$B = A \cup (B \cap A^c)$, so from Axiom 3:

$$P(B) = P(A) + P(B \cap A^c) \geq P(A).$$